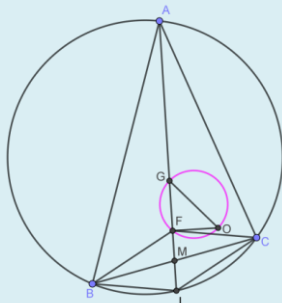


Author's Solution

Before going to the solution, let us know some properties of “The Eight points circle Theorem”, explained in pages 44 to 48 of the book “ Raja Climax’s Theorems on Geometry” available in this website.



$\triangle ABC$ is inscribed in the circle. O is its orthocenter and G is its centroid. The circle drawn using GO as diameter is the Eight Points Circle of the $\triangle ABC$.

The Eight points circle Theorem, tells the following:

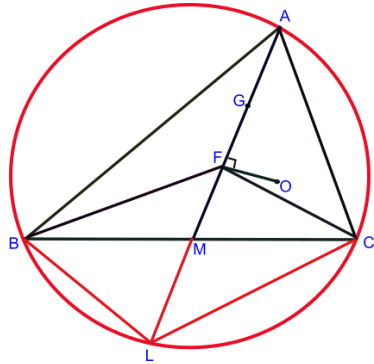
- 1) Every triangle has a distinct eight points circle.
- 2) The diameter of the Eight Points Circle is the line segment joining the orthocenter (O) and the Centroid (G) of the triangle.
- 3) The Eight Points Circle intersects the median at two points.
 - a) One at the Centroid and
 - b) The other at the point whose distance to the midpoint of the side (along the median) is equal to the distance of the said midpoint to the circumcircle. (Let AM produced meet the circle at L).
i.e., $FM = ML$ (see picture)

4) Since GO is the diameter of the Eight Points Circle, $\angle GFO = 90^\circ$
 In other words, $OF \perp AM$.

Now, let us take up the solution

Construction :

Draw the circumcircle of the $\triangle ABC$.
 Produce AM to meet the circumcircle at L . Join FB , FC , LB & LC .



Proof:

As per 4 & 3 (b) above,
 $OF \perp AM$

And $FM = ML$

$BM = MC$ (given)

And $FM = ML$

\Rightarrow In the quadrilateral $FBLC$, the diagonals FL & BC bisect each other.

\therefore $FBLC$ is a parallelogram

$BL = FC$ -----(1)

And $BF = LC$ -----(1)

As per the Twelve points circle theorem contained in pages 39, 40 & 41 of the book **The Raja Climax's Theorems on Geometry**, the chord cut out by the **Nine-Point Circle (Twelve points circle)** on a median is equal to half the extended median.

$\Rightarrow MG = \frac{1}{2} AL$

As per Sai Kumar Theorem (Page No: 31 of the book **The Raja Climax's Theorems on Geometry**),

$$2AL^2 = AB^2 + BL^2 + LC^2 + CA^2 \text{----- (3)}$$

(1), (2) & (3) →

But $AL = 2MG$

$$\therefore 2AL^2 = 8MG^2 \text{-----(4)}$$

(1),(2) & (4) →

$$8(MG^2) = AB^2 + AC^2 + FB^2 + FC^2$$

-----Hence Proved

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